

Final Project

Teaching Philosophy Statement

I love mathematics. I find it to be one of the most useful subjects in everyday life. Math is also beautiful. Math has a structure that shows up in so many places, including things like how the Fibonacci sequence exists in flowers, tessellations and transformations in structures and art, fractals in the neurons of our brain, the concentric circles that form when dropping a pebble into a pond. I love helping students to find the joy in math.

When it comes to what mathematics is, I believe that math is a study of patterns, structures, and relationships that allow us to explain and learn about the world around us. It is a discipline where ideas are used to transform information into something that is not known, or to be able to think about things in a new way. It is not just memorizing equations and formulas, and finding a specific answer at a specific time, as many students seem to view and experience math. It is an ever-changing thing, evolving and interconnected.

One of my main goals as a mathematics educator is to help students become mathematical thinkers—not just being able to understand the content, but who see themselves in the mathematics that we do together, and see themselves as capable, resilient and curious problem-solvers. I want my classroom to be a place where all students, especially those who are historically not represented, can feel a sense of belonging and have ownership over their learning. Students will leave my classroom knowing that they are mathematicians, and that through struggle, our brains learn and grow. Even if they do not use the formulas that we learn every day, they will be able to use the ideas of problem-solving and the resiliency that mathematics can teach in their lives.

Problem-solving directly impacts how students become mathematical thinkers. In order to engage in mathematics at its truest form, students must problem solve. Through problem-solving, students are able to learn about math, while developing strategies, learning to justify their reasoning, reflecting on what they have already done, and planning where they would like to go. They are able to take risks, identify which ideas have worked, and what needed some refinement. Mistakes are not something to be ashamed of, but instead something that they used to learn more about the problem. All of these things are exactly the ideas and thoughts that I want my students to be able to leave my classroom with.

Problem-solving allows students to deepen their understanding, because they are more in charge of constructing their own knowledge. It also allows me to learn more about how students think, and what they understand, and even more importantly, what they might not yet. I want students to know that I value the process of learning. Problem-solving allows for students to not just focus on the correct answer, but the journey to get there.

From the models that we have studied, the one that I really connected to was Schoenfeld's four category framework. The four frameworks—resources, heuristics, control and belief systems—do go hand in hand with my goals for my classroom, especially control and belief systems. Control surrounds the ideas of how students are able to plan their solving, monitor and assess themselves throughout the process, and how they know when to abandon or tweak a plan as they solve. This is the essence of metacognition, and how I want students to be able to not only develop mathematical tools but also know how and when to use them. This is something that I try to model by thinking aloud during working on problems or asking

students to explain if different aspects make sense, or even if there are better ways to approach the problem.

Along with control, I fully think that the “belief systems” component is central to students’ thinking about mathematics. Students’ beliefs about both themselves and math play a huge role in how they engage with the subject. Many students don’t believe that they are “math people” and that they are not good at math and can never be good at math. This is a conception that needs to change. If students do not believe that they are capable, it is much harder for them to be able to engage with the content. I want to work with students to help them to see themselves and to honor their growth. I want them to focus more on their effort and progress, rather than if they got it fully correct the first time. My hope is that, over time, students begin to see themselves as people who truly belong in mathematics.

Distributed Problem-Solving Activities

One of the courses that I will be teaching next fall is Intermediate Algebra. This is what my school calls Algebra 1. This course consists of typically 9th grade students, but often there are many sophomores retaking the class as well, as this is a class that students must pass to graduate. In my school, over 78% of our school population qualifies for Free and Reduced lunch. Additionally, about 40% of our students do not speak English as their first language. We also have a large number of students who receive special education services. Overall, many students also come in far below grade level in terms of Mathematical ability. In Intermediate Algebra, we work on standards including systems of equations, exponents, factoring, polynomials, and quadratic equations.

For this assignment, the framework that I chose was the Distributed Problem-Solving Activities Option. I wanted to include problem solving activities in more places in my course, so I felt this option gave me the most freedom to do this. I wanted to integrate problem solving where it naturally supports student thinking, especially at spots where I see misconceptions commonly arise, or where students can think further. Some of the locations that I felt could benefit the most from problem-solving activities were solving quadratics through factoring, setting up equations from context problems, and interpreting graphs.

Objective 1: Equations from Context

The first learning objective that I selected was having students set up equations from context problems. At a minimum, students should be able to identify key quantities involved, distinguish relevant information from extraneous details, and represent these relationships using a linear equation (both one and two variables). Students should be able to produce an equation independently when given a well-scaffolded word problem and explain the meaning of each part of their equation. More aspirationally, I want students to be able to transfer this skill across different types of functions, including exponential and quadratic equations, as well as being able to create meaningful context for equations that they are given.

Students often struggle with these problems, no matter what algebraic concept we are covering. Some students have developed such a fear of context problems by ninth grade that the minute they see a problem with text, they do not even attempt it, even if they have a solid foundation of math knowledge. I often see students struggle to translate the ideas into an explicit equation. Students may be able to identify what the end result should be, as well as talk

about what values they have found when it is discussed out loud, but translating into an equation still proves to be a struggle. Other students cannot identify what they end result should be, or what parts of the problem are important. I often see students stumble into a way of thinking, where they reason that if it the problem contains a number, it must be put into the equation.

To help students feel more confident when it comes to creating equations, I would like to incorporate some problem-solving activities to help with this. This would take place in the systems of equations unit, where most of the work on context problems and graphing in Intermediate Algebra takes place. Students will already have been exposed to different graphing concepts, such as rates of change and intercepts. Additionally, we will have done work in a prior chapter about being able to look at a linear graph and describe information about the topic, such as looking at the speeds of a taxi-cab driver and being able to articulate what is happening; for example, when the speed is zero, the driver is stuck in traffic.

To begin the activity, I would give students equations and different contexts and have them match the models to the contexts. Depending on the class and students, they would be analyzing three to four different sets of equations and contexts. Students will begin by working on their own to try to develop their own thoughts and ideas about which equations match the different contexts. As they are working through this first part, they will be asked to write down their thinking in a journal, much like we did on the problem-solving set this semester. This recording and journal can be scaffolded for students as needed, including sentence frames, word banks, voice recordings, or other tools. This is something that in general I would like to

include in my class on a more frequent basis, so by the time we reach this unit, we would have already done some lower risk attempts with this recording process.

After students have gotten a chance to work through these equations and contexts on their own, they would come together as a small group of three to four students. They would discuss their thinking with a group and come to a group conclusion on what contexts and which equations match. After each group has determined the matching, they will pick one set of these matched equations and contexts. This set of equations and context will be close enough that they can determine that they match, but the equations themselves will be incorrect. For example, a problem about the prices of two different cell phone plans might be correctly represented by the equation $y = .50x + 20$ and $y = 0.25x + 40$, but the sample student has written different equations, maybe putting variables in the wrong spots or using a decoy value from the problem. The sample student will also have written an incorrect justification for their choices.

Within their groups, students will be asked to identify any errors in the equation and in the sample student explanation. Students would then need to revise the equation and then justify what the correct model is. This is a clear way to help students make sure that they are understanding the problem. Some techniques that students could use to verify that they understand what the problem is asking, include using different techniques such as constructing a table, guess and check, and sketching drawings or diagrams to help them break down the problem.

As students are working through this activity, they are definitely developing some ideas of control, as they are needing to monitor and evaluate their mathematical reasoning, as well as using their resources to be able to develop tables or diagrams to justify their understanding. I

have found that when students are analyzing other people's errors instead of their own, they tend to feel more comfortable engaging critically and reflectively.

As students are going through this activity, if they are exceeding the minimum expectations, I would encourage the groups to create a context for another group to create an equation for, but also needing to give them an incorrect answer and justification, essentially creating a student sample that another group would be able to find an error in. However, they also need to be able to create the "answer key", therefore having a correct equation and justification. It should be complex enough that it would be tough for another group to be able to solve, which may encourage them to start getting into ideas of systems of inequalities.

Objective 2: Interpreting Graphs

The second learning objective that I selected was interpreting graphs. At a minimum, students should be able to identify (and label) the x- and y-axes in context, including units. Additionally, I want students to be able to describe and interpret x- and y-intercepts, various other points, and the rate of change from a linear graph and explain what these values mean. It is important to me that students can describe general trends, such as when the graph is increasing or decreasing or has a constant change. Students should also be able to estimate values from the graph. For the aspiring goal, I want students to be able to work with various function types such as quadratic, exponential, absolute value. I want them to be able to transfer skills that we are working with such as symmetry and what that would look like on a graph and explain how the slope changes in a quadratic equation or exponential versus a linear equation.

Students usually can do very well with plotting points on a graph given an equation. However, when given a graph, students often struggle to convey ideas about the information presented. First, students can forget to read the scale of a graph or misidentify the axes. This means that when they are attempting to find the slope of a graph, they can get incorrect interpretations about what is being shown. Translating specific points into context is also often a struggle for students. When identifying what is a solution to a context based on the graph, they struggle with conceptualizing that there can be multiple solutions to a single problem. I also see students consistently mixing up the coordinate points, which leads to incorrect assumptions and answers about the graphs.

The activity for this objective would take place in the end of the review unit on eighth-grade graphing concepts. My department has identified that this is a place that students often need to revisit, so by placing it toward the end of the unit, students will have gotten a chance to review the concepts that they often forget or didn't learn the first time around. This review unit covers topics like slope, intercepts, increasing vs. decreasing intervals, and thinking about the graphs in a context. Before the start of this activity, we would already have done some scaffolded work with these types of graphs. For example, students would have already been given graphs and have needed to decode the features, like what intervals are increasing and decreasing, and how those matched up to a scenario.

To begin this activity, students would be given a short context, such as tracking distance over time. On a graphic organizer, students would begin by reading the context and then identifying key points and important information. This would include things like if the graph should be increasing or decreasing, the rate of change or intercepts, or the intercepts. They

would then work with a partner who had the same context (I would identify these partners ahead of time), to then verify their information. These two students would then create a plan for what items would need to be added to a graph and in what order. On the same graphic organizer, they would write out this plan. It should be detailed enough for another student to follow. The plan creation, identification of key information, and even the contexts can be scaffolded for students as needed.

After the partners have created their plan, they would pass this plan and context off to another group, possibly identified by me ahead of time. This next group would then follow the plan, and verify that they correctly executed the graph, as well as identifying if the original group that created the plan missed any important aspects. They would then reflect on how they were able to follow the plan, as well as reflecting on the original group's ideas. This second group would then again pass off the context and the plan to another group, who, starting from scratch, would complete the same process. After two to three rounds of this, all graphs and reflections would go back to the original group. They are able to then reflect on if other groups were able to complete their plan, as well as identify where their can change or update their thinking for next time.

A way that students can extend their thinking with this activity would be to have them create a completely new context that would match the graph they ended up with. This forces them to really be able to articulate what the graph shows and how to describe these situations into real-life. Just saying that the slope is $\frac{4}{9}$ doesn't help when talking about the price of cookies.

This activity has students thinking out loud, as well as breaking down the problem into parts, because some things must be finished before others, such as drawing the scale before plotting the points. Student both need to create and carry out a plan, as well as to monitor and assess while working through other students work. They both need to be able to explain what they have done, as well as be able to explain when they deviated from the plan or revised it. Students are also each contributing to a larger body of work, in which they are helping to develop their own mathematical worldview.

Objective 3: Solving Quadratics through Factoring.

Finally, the last learning objective that I selected was solving quadratic equations, specifically using factoring. At a minimum, students should be able to factor quadratic equations of the form $ax^2 + bx + c = 0$, where $a = 1 \wedge a \neq -1$, and use the zero-product property to find real-number solutions. For a student moving to an aspire level, I would like for students to be able to consistently recognize special cases of trinomials, specifically perfect square trinomials and apply shortcuts to these types of trinomials. Additionally, students can start to find solutions for trinomials with non-real solutions, or trinomials with a greatest common factor that contains an x .

This is a topic that I see students struggle with often. First, I feel that students don't have a good conceptual understanding of what those solutions mean. Students may memorize that they are x-intercepts, but why do some quadratics have no solution? What do those x-intercepts mean in the context of a problem? Additionally, students often fail to recognize when problems contain a greatest common factor or a leading negative, which leads to more problems later on.

The traditional methods for factoring contain a majority of guess-and-check strategies, which can be very frustrating for students as they don't see the end in sight. Another thing that can make factoring tough for students is that they don't have a concrete understanding of how factoring is the opposite of multiplication of binomials. Students often struggle with their multiplication and addition facts. If students do not have a good foundation of basic math facts, it can be a large struggle for them to find factors of number that multiply and add to give specific values.

When it comes to an activity idea for solving by using factoring, I wanted to begin by having students work backwards, by giving them what the solutions to the quadratic would be and then having them figure out what the original quadratic was. Students may identify that there are multiple equations that satisfy this requirement, which can lead to excellent discussion. More than likely though, students would construct this equation by multiplying the roots back together. This would come after the first half of the quadratics unit, so students would already be familiar with ideas about axis of symmetry and what the solution to the quadratic is on an already graphed quadratic. It also comes after polynomials, so students would be familiar with multiplication of binomials. There is a second mini unit on quadratics after factoring, where we talk about graphing and solutions and how to pick the best solving method—factoring vs. completing the square vs. quadratic formula. This activity would be coming inside the factoring unit.

Student would begin with a quadratic in factored form and use multiplication to determine what the original quadratic equation looked like in standard form (starting with $a = 1$). I would then have students use their graphing calculator or other software such as Desmos

to graph their solution and identify the x-intercepts of the graph. They can also verify if they multiplied correctly by graphing both equations. What I want students to start to see is that finding the solutions is the same as having the factored form. Eventually, students will be given more and more information, such as if there were a greatest common factor. This provides students a context for factoring, which is something that lots of students lack.

For the second part of the activity, students are going to move in the opposite direction. Students are going to be given a few standard form quadratics. There will also be the inside of the boxes for “box-multiplication” which will be filled in for students. Working in a group for this part of the activity, students will begin to determine what the outside of the boxes needed to be (what the two binomials that were multiplied together to give the inside numbers). With their group, I want students to articulate in their problem journals what they notice about the specific problems and if they can identify how the values inside of the boxes come about. Eventually, the groups hopefully reach the conclusion that in the top left square is the ax^2 term, in the bottom right corner is the c -value, and the two diagonal values add up to give the bx -term. Along the way, I will support students with questioning and asking guiding questions if they need it. For extension opportunities, I would give students trinomials that don’t factor with integers but have them start with the graph first. I want to challenge students to determine if there are reasons that some graphs don’t have x-intercepts. How does this relate to the factoring process?

By working backwards, students are able to leverage a concept that they understand, multiplication, and determine how it specifically relates to factoring, which is a key concept that students miss. As students begin working with standard form trinomials and identifying what

parts go in the box when beginning to factor, students are using many of their mathematical resources, including factors of numbers, addition and multiplication facts, and multiplication of binomials.

Additional Practices

In addition to these specific activities, there are some broader practices that I want to use to support students' growth as problem-solvers. One of the main practices that I will introduce more consistently is the use of reflection and metacognitive journals. I want to give students regular opportunities to reflect on their thinking during and after tasks. I want students to consider what strategies they used, where they got stuck, what worked, and how they have grown in their learning. I also really loved the idea of wrappers on assessments and assignments. Having students reflect before they start a task, and after, evaluate what worked, what didn't, and how they might approach similar problems in the future. This helps students to make connections between effort, strategy, and performance. Additionally, it helps students to see that they are growing and learning. Lastly, I want to emphasize the importance of making mistakes. I think that by doing more work with error analysis and asking students to analyze incorrect solutions (either their own or sample student work), identify what went wrong, and explain the correct reasoning, will allow for students to see that revision is a vital part of the mathematics process. Overall, I hope to support all students in building stronger mathematical identities and approaching problem-solving with more confidence and resilience.