Homework #1 - Math 626

#Math 626/Module 0

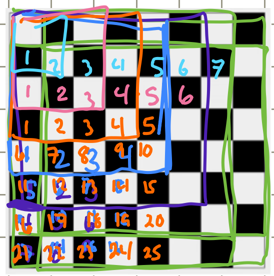
1) First A problem

A1. How many squares can be seen on a standard 8 x 8 chessboard? [Hint: the answer is much larger than 64.]
How many squares can be seen on an n x n chessboard for **any** n > 0?

8×8 squares: 1 $=1^2$
7×7 squares: 4 $=2^2$
6×6 squares: 9 $=3^2$
5×5 squares: 16 $=4^2$
4×4 squares: 25 $=5^2$
3×3 squares: 36 $=6^2$
2×2 squares: 49 $=7^2$
1×1 squares: 64 $=8^2$

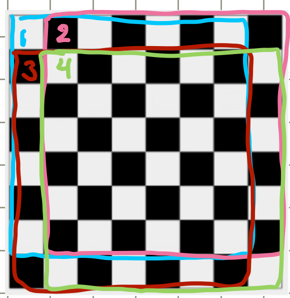
Total Squares: 204

$$\sum_{i=1}^{8} i^2$$



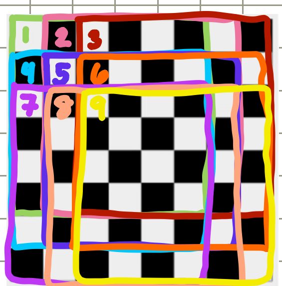
For any $n>0$, the # of squares on an $n\times n$ chessboard is $\sum_{i=1}^{n} i^2$

Explanation of steps: I began by looking at the 8×8 chessboard, and finding squares of size 8×8, of which there was 1.

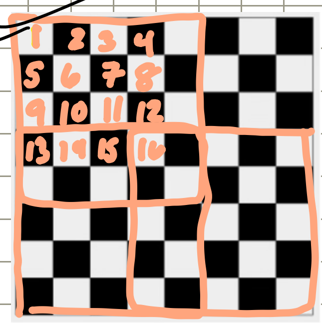


Then I looked at squares of size 7×7. There were 4 of them.

For squares of 6×6, there were 9 of them. Here is where I started to see a pattern. looking at the corners, it created a 3×3 square, so there were 9 possible corners.



I verified with the 5×5 square, which did have 16. Similarly with the the 4×4s (25), 3×3s (36), 2×2s (49), and 1×1s 64.

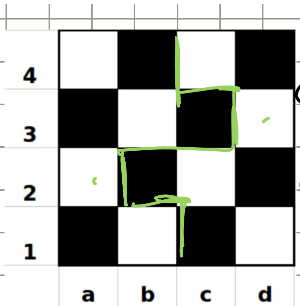


To find the total number, you add all of these values up. Each of these is a perfect square, So for an 8×8 square, the Total squares is $\sum_{i=1}^{8} i^2 = 204$. This is the pattern for any $n\times n$ square.

2)

2nd A type

A4. In how many different ways can a 4x4 chessboard be divided into **two** pieces by cutting along the solid lines that form the squares in such a way that the resulting pieces are the same size and shape (that is, so that they can fit on top of each other using **only *translation* and *rotation***)? Note that we only care about the size and shape of the resulting pieces: cutting the board along the vertical line between column b and c and cutting it along the horizontal line between rows 2 and 3 would both be considered to be **valid** but are not **distinct** solutions.



4,4,0,0 ✓
4,3,1,0 ✓
4,3,0,1 ×
4,2,2,0 ✓
4,2,1,1 ×
4,2,0,2 ×

4,1,3,0 ✓
4,1,2,1 ×
4,1,1,2 ×
4,1,0,3 ×
4,0,.... ×

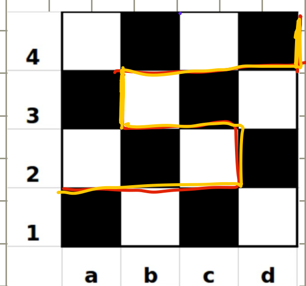
Middle Rows can't be 0 or 4.

3,3,2,0 ×
3,3,1,1 ✓ match 4,2,2,0
3,2,3,0 ×
3,2,2,1 match 4,3,1

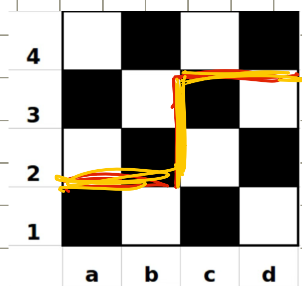
3,2,1,2 ×

outsides add to 4 insides

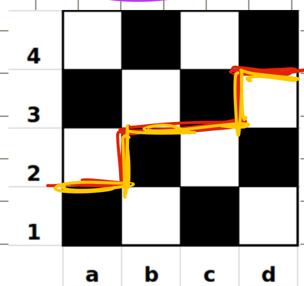
4,1,3,0 ✓
4,2,2,0 ✓
4,3,1,0 ✓
3,3,1,1 ✓
3,2,2,1 ✓
3,1,3,1 ✓
2,3,1,2 ✓
2,2,2,2 ✓ Same
~~2,1,3,2~~



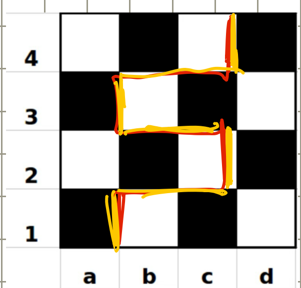
4,1,3,0



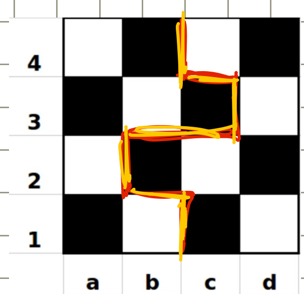
4,2,2,0
also 3,3,1,1



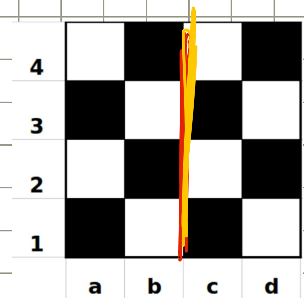
4,3,1,0
also 3,2,2,1



3,1,3,1



2,3,1,2



2,2,2,2

6 ways

Explanation of Steps: I started by trying to find solutions by brute force. I then used Ellen's suggestion from the discussion boards to find the # of cells in each row. It was easy to see that if either one of the inside rows must not be zero or it will break into more than 2 pieces. Each piece must also have 8 squares. If the middle rows were 4, it wouldn't be symmetrical (Aside from the case of slicing in half).

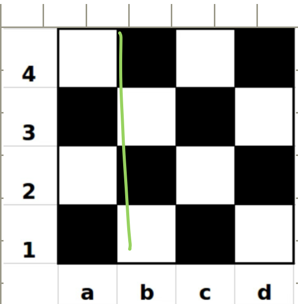
Some options didn't work, (4,2,1,1) (3,2,1,2) (4,1,2,1) as they didn't produce symmetrical pieces. Examining the cases that worked, it was easy to see that the outside rows added to 4, as well as the inside rows. This is because to form the symmetry, there can't be more than 4 in the corresponding rows.

After finding all options, I drew them on the chess board. Some cases were the same, just rotated (not distinct). There were 6 total options

3)

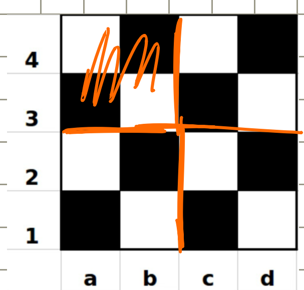
B-Extension

B4. In how many different ways can a 4x4 chessboard be divided into **four** pieces by cutting along the solid lines that form the squares in such a way that the resulting pieces are the same size and shape (that is, so that they can fit on top of each other using **only *translation* and *rotation***)? (e.g. combining the horizontal the vertical cuts discussed above creates four equivalent 2x2 quarters – is this the only way?)

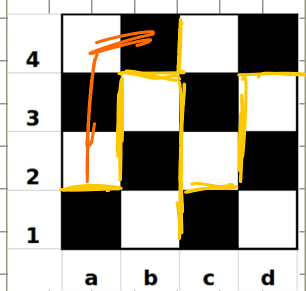


4,0,0,0
~~3,1,0,0~~
2,2,0,0 ✓
2,1,1,0 ✓
1,2,1,0
1,1,2,0 ✓
1,1,1,1

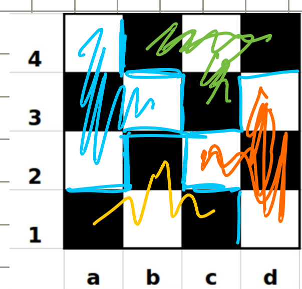
Each piece has 4 squares
↳ no inside 0's (aside from case 1-3)



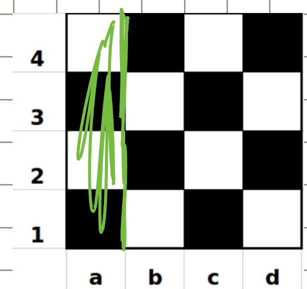
2,2,0,0



2,1,1,0
also 1,1,2,0



1,2,1,0
3,1,0,0



1,1,1,1
4,0,0,0

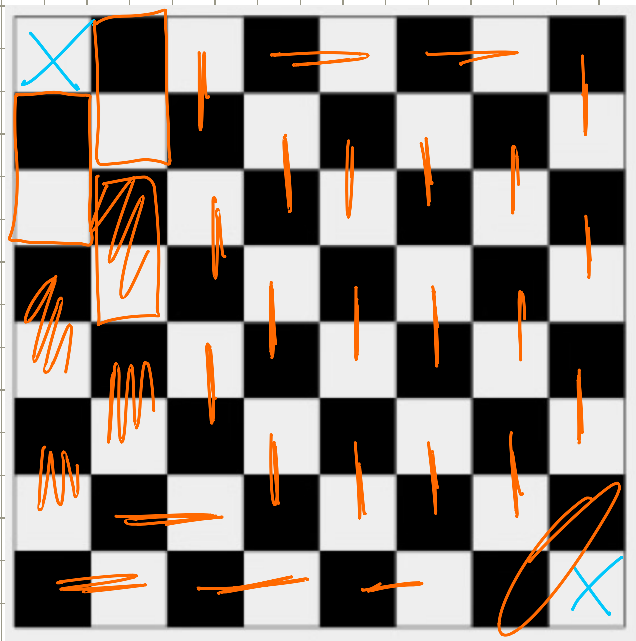
Explanation of steps: I apprached this much the same way as the previous problem. Each piece must have 4 squares, Since there were fewer cases, it was easier to check these case by case.

Additionally, I verified my findings by looking at A4. All shapes had to be the same, so If I broke the shape in A4 into 2 equal parts, it needed to match a shape from B4.

Overall, there were 4 different ways to slice the board.

4) 3rd A problem

A2. Suppose two opposite corner squares are removed, leaving a truncated board with 62 total unit squares. Determine whether or not it is possible to cover the remaining squares using 31 1x2 dominoes.

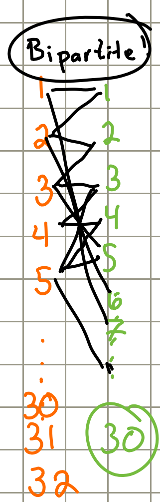


It is not possible.

Explanation of Steps: I started by laying out dominos to see if it was possible. It seemed to not work.

As I continued testing out placements, I noticed that each domino would have to go on one white block and one black block. As there are two black blocks left over, there is no configuration that would work.

Additionally, It seems as though it ends up being a bipartite graph. It can be separated into two blocks (white squares and black square) and every black square cannot connect to another black, and same for white.



However, there are two extra black vertices. So, when every white vertice is connected, there are two black nodes that won't and cannot connect.