Homework #1

Pictures for #1 at end of Homework.

Math581
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1. Icosahedron

Faces = 20
Edges = 30
Vertices = 12

F - E + V =
20 - 30 + 12 =
-10 + 12 = $\boxed{2}$

Torus

Faces = 12
Edges = 24
Vertices = 12

F - E + V =
12 - 24 + 12
-12 + 12 = $\boxed{0}$

Tori: Extrapolations

	Faces	Edges	Vertices	Euler's formula
1 Torus:	8 slanted 4 outer Box 12 total	4 - inner square 8 - outer box both sides 4 - connecting sides 8 - slanted 24 total	4 - inside square 8 - outside box corners 12 total	= 0
2 Tori:	Adding another set of slanted faces for each additional Tori +8 slanted 16 slanted 4 outer box 20 total	+ 1 set inner square (+4) + 1 set slanted (+8) + 1 set connecting Tori (+2) +14 8 - inner square 8 - outer box 4 - connecting sides 16 - slanted + 2 - connecting the tori 38 edges	+ 1 set inside squares (+4) 8 - inside squares 8 outside box corners 16 total	F - E + V = 20 - 38 + 16 = -18 + 16 = -2
3 Tori:	20 from 2 Tori + 8 slanted 28	38 from 2 Tori +14 new 52	16 from 2 Tori + 4 new 20	F - E + V = 28 - 52 + 20 = -24 + 20 = -4
4 Tori	28 + 8 36	52 +14 66	20 + 4 24	36 - 66 + 24 = -30 + 24 = -6
5 Tori	36 + 8 44	66 +14 80	24 + 4 28	44 - 80 + 28 = -36 + 28 = -8

→

1 cont. n Tori

↳ every time a Torus is added, the answer for Euler's formula goes down by 2 each time.

1 torus = 0
2 tori = -2
3 tori = -4
⋮

$$f(n) = -2n + 2$$

$f(1) = -2(1) + 2 = -2 + 2 = 0$ ✓
$f(2) = -2(2) + 2 = -4 + 2 = -2$ ✓
⋮

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2. Let $f(x) = 4x - 9$. Prove – If $x_1 \neq x_2$, then $f(x_1) \neq f(x_2)$.

Contratposition: If $f(x_1) = f(x_2)$, then $x_1 = x_2$.

Proof: Assume $f(x_1) = f(x_2)$. Thus, $4x_1 - 9 = 4x_2 - 9$. Thus, $4x_1 = 4x_2$ and $x_1 = x_2$. So, we can conclude that if $x_1 \neq x_2$, then $f(x_1) \neq f(x_2)$ ■ :)

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3. Proof: Suppose that n and m are odd. Thus, $n = 2k+1$ and $m = 2l+1$ for some $k, l \in \mathbb{Z}$.

So, $m \cdot n = (2k+1)(2l+1)$
$= (4kl + 2k + 2l) + 1$
$= 2(2kl + k + l) + 1$.

Thus, by def. $m \cdot n$ is odd ■ :)

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4. Scratch Work

Let $\varepsilon > 0$. We want $11 - \varepsilon < f(x) < 11 + \varepsilon$

$$11 - \varepsilon < 3x + 5 < 11 + \varepsilon$$
$$-5 \qquad -5 \qquad -5$$
$$\frac{6 - \varepsilon}{3} < \frac{3x}{3} < \frac{6 + \varepsilon}{3}$$
$$2 - \frac{\varepsilon}{3} < x < 2 + \frac{\varepsilon}{3}$$

Proof: Let $\delta = \frac{\varepsilon}{3}$ and $\varepsilon > 0$. Assume $2 - \delta < x < 2 + \delta$, then

$$2 - \frac{\varepsilon}{3} < x < 2 + \frac{\varepsilon}{3}$$
$$6 - \varepsilon < 3x < 6 + \varepsilon$$
$$11 - \varepsilon < 3x + 5 < 11 + \varepsilon$$

Thus, if $2 - \delta < x < 2 + \delta$, then $11 - \varepsilon < f(x) < 11 + \varepsilon$ ■ :)

5. If $x > 0$, then $x^2 - x \geq 0$.

Consider $x = \frac{1}{2}$.

$\frac{1}{2} > 0$, but $\left(\frac{1}{2}\right)^2 - \frac{1}{2}$

$= \frac{1}{4} - \frac{1}{2}$

$= -\frac{1}{2} < 0$

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6. Proof (by cases)

Assume $n = 2k$, for some $k \in \mathbb{Z}$ such that $n > 0$.

Thus $n^2 + 3n + 8 =$

$= (2k)^2 + 3(2k) + 8$

$= 4k^2 + 6k + 8$

$= 2(2k^2 + 3k + 4)$.

So, by definition $n^2 + 3n + 8$ is even when n is even.

Now, assume $n = 2k+1$ for some $k \in \mathbb{Z}$ such that $n > 0$.

Thus, $n^2 + 3n + 8 =$

$= (2k+1)^2 + 3(2k+1) + 8$

$= 4k^2 + 4k + 1 + 6k + 3 + 8$

$= 4k^2 + 10k + 12$

$= 2(2k^2 + 5k + 6)$.

So, by definition, $n^2 + 3n + 8$ is even when n is odd.

Therefore, we have shown that for every positive $n \in \mathbb{Z}$, $n^2 + 3n + 8$ is even. ∎

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7. $U = \{1, 2, \ldots, 10\}$, $A = \{1, 3, 5, 7, 9\}$, $B = \{2, 4, 7, 9\}$, $C = \{4, 7\}$

a) $A \cap B = \boxed{\{7, 9\}}$

b) $A \cup B = \boxed{\{1, 2, 3, 4, 5, 7, 9\}}$

c) $A \setminus (B \cap C) = \boxed{\{1, 3, 5, 9\}}$

$B \cap C = \{4, 7\}$

d) $A \setminus C' = \boxed{\{7\}}$

$C' = \{1, 2, 3, 5, 6, 8, 9, 10\}$

e) True or False: $C \subset (A \cap B)$.

$(A \cap B) = \{7, 9\}$

is $\{4, 7\} \subset \{7, 9\}$? $\boxed{\text{False}}$

$4 \in C$, but $4 \notin A \cap B$, so C is not a subset.

f. True or False $C \setminus B = 7$

$C \setminus B = \{\}$ $\boxed{\text{False}}$

$C \setminus B$ is the empty set since both 4 and 7 are in B.

8. Proof Suppose that $A \subseteq (B \cap C)$ and $(B \cup C) \subseteq A$. To prove that $A = B$, then we must show that $A \subseteq B$ and $B \subseteq A$.

First, we need to show that $A \subseteq B$.

Let $x \in A$. Because $A \subseteq (B \cap C)$, then $x \in B \cap C$. So, $x \in B$ and $x \in C$.

Thus, we've proved that $A \subseteq B$.

Now, we will show that $B \subseteq A$.

Let $x \in B$. Since $(B \cup C) \subseteq A$, we know that $x \in A$.

Thus, we've proved that $B \subseteq A$.

Since $A \subseteq B$ and $B \subseteq A$, we've proved that $A = B$ ■ ‿

9. $A_n = (1+\frac{1}{n}, 3]$ for each $n \in \mathbb{N}$.

a) $\bigcap_{n=1}^{\infty} A_n = (2, 3]$

Proof

Let $x \in \bigcap_{n=1}^{\infty} A_n$. Thus, there exists $n \in \mathbb{N}$ such that $x \in (1+\frac{1}{n}, 3]$.

By the Archemedan Property, $1+\frac{1}{n} < x$, so x is always bigger than 1. However, lets examine when $n = 1$. Since $1+\frac{1}{n} < x$, $1+\frac{1}{1} < x$, so $1+1 < x$, and $2 < x$. Therefore $x \in (2, 3]$, so $\bigcap_{n=1}^{\infty} A_n \subseteq (2, 3]$.

Now, we need to show that $(2,3] \subseteq \bigcap_{n=1}^{\infty} A_n$.

Assume $n = 1$. Since $A_n = (1+\frac{1}{n}, 3]$, then when $n = 1$ is $(2, 3]$.

We know that $(2,3] \subseteq \bigcap_{n=1}^{\infty} A_n$ because it is one of the sets in the intersection.

So, $\bigcap_{n=1}^{\infty} A_n = (2, 3]$ ■ ‿

9b) $\bigcup_{n=1}^{\infty} A_n = (1,3]$

First, we need to show that $\bigcup_{n=1}^{\infty} A_n \subseteq (1,3]$. Let $x \in \bigcup_{n=1}^{\infty} A_n$. Therefore, there exists a $n \in \mathbb{N}$ such that $x \in (1+\frac{1}{n}, 3]$. Because $n \in \mathbb{N}$, then $0 < \frac{1}{n}$, and $1 < 1+\frac{1}{n}$. Therefore $1 < x$. So, x must be an element of $(1,3]$, proving $\bigcup_{n=1}^{\infty} A \in (1,3]$

Now, Assume $x \in (1,3]$. Therefore, there exists an $n \in \mathbb{N}$ such that $1+\frac{1}{n} < x$. This shows that $x \in (1+\frac{1}{n}, 3]$. So, we know that $(1,3] \subseteq (1+\frac{1}{n}, 3]$, and thus $\bigcup_{n=1}^{\infty} A_n = (1,3]$ ■ :)