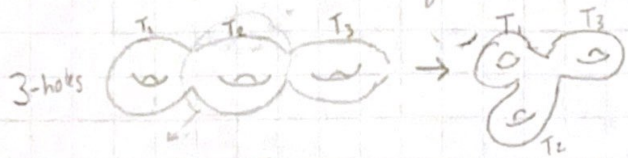
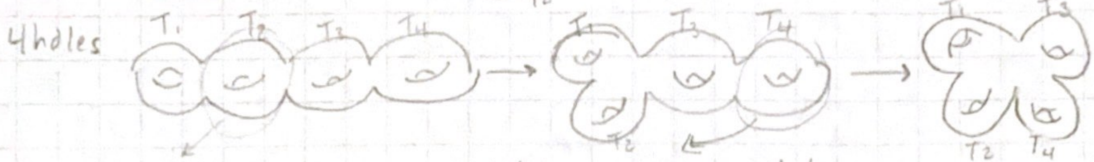
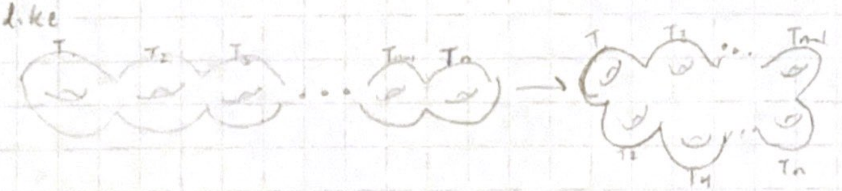
Homework #14

1) Show a torus with n holes in a line is homeomorphic to a n-holded torus with the holes arranged in a circle.

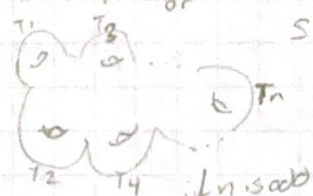




So with n-holes, we can just take every other hole + stretch and move it down like



if n is even or



if n is odd

Then you can kind of just stretch everything out a bit more to make it circular shaped.

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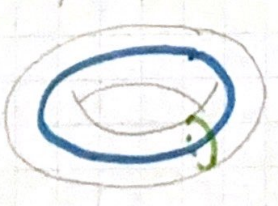
2) Why is $S_1 \times S_1 \cong \mathbb{T}^2$



With the Torus, imagine the blue line is one of the S_1 and the green line is another one of the S_1.

Essentially $\mathbb{T}^2$ is a circle of circles or $S_1 \times S_1$

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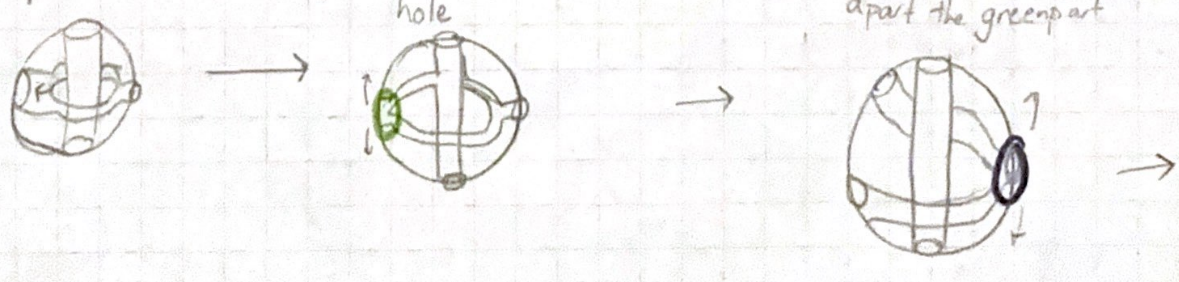
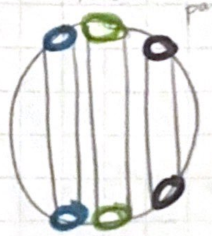
3. The hole in a hole in a hole is homeomorphic to a 3-holed torus.

If we start with the hole^3, then we stretch the inside hole. Then we pull apart the green part



Then do the same thing on the other side

Then you can move the tubes to be parallel



then you can flatten and stetch it into a Three holed torus

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4. We know there are two types of 1-manifolds - lines and circles. Since we are looking at a compact + connected 1-manifolds, it would be the Circle, S_1, because the $\mathbb{R}^1$ line is not connected because it is not bounded, and the S_1 circle is bounded.

5. This means that compact 1-manifolds are the $\mathbb{R}^1$ line, because they are not bounded.