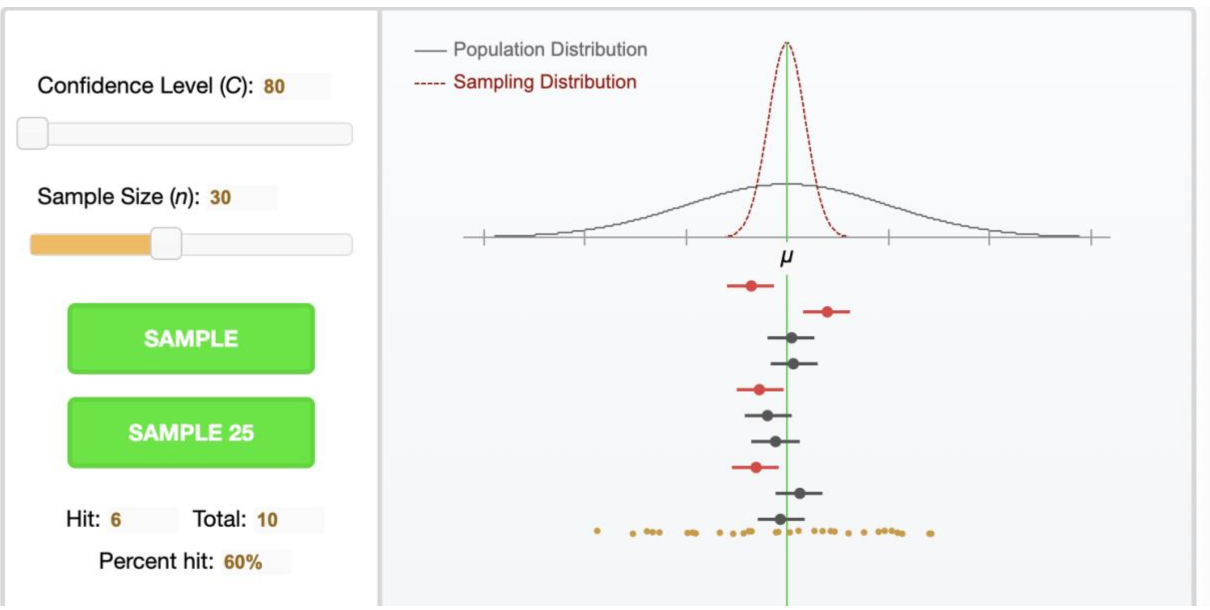
Madeline Kuehn
MATH 634

Reflection #5: Statistical Inference Using Confidence interval and Hypothesis Testing: Single sample mean and single proportion reflection.
Part 1

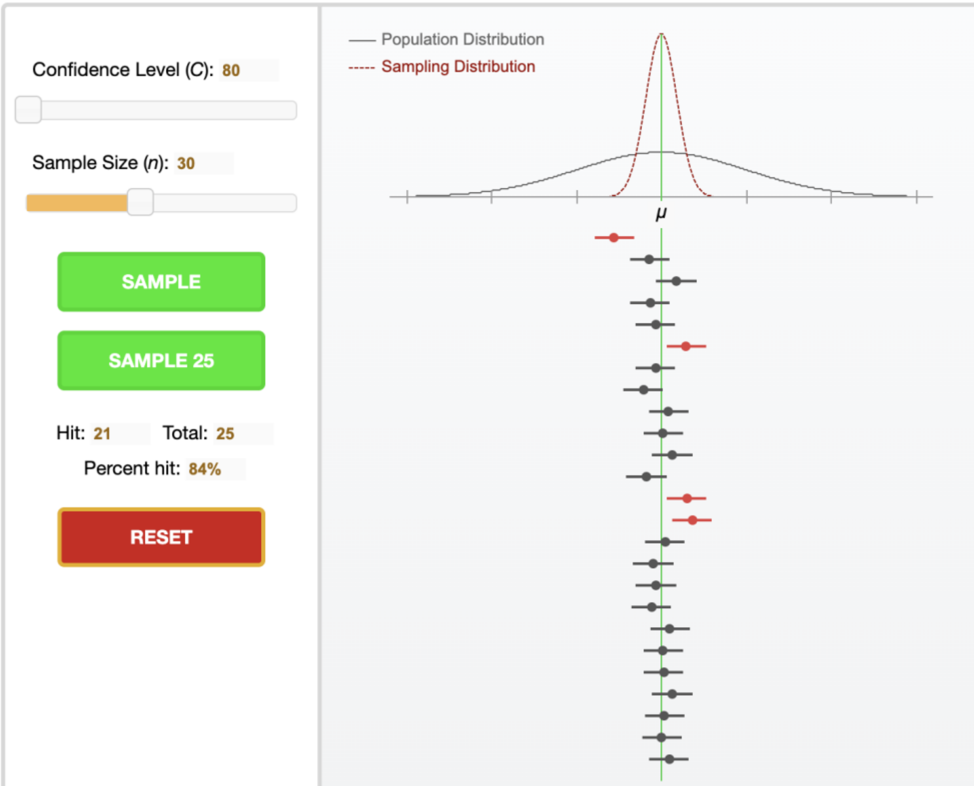
1. The idea of an 80% confidence interval is that, in 80% of all samples, the method produces an interval that captures the true parameter value. That's not a high confidence for practical use, but 80% hits and 20% misses make it easy to see how a confidence interval behaves in repeated samples from the sample population. Click on the link below to take you to Confidence Interval Applet. http://digitalfirst.bfwpub.com/stats_applet/stats_applet_4_ci.html

a) Set the confidence level to 80% and the sample size to 30 using the sliders. Click on "Sample" to select Simple Random Sample (SRS) and calculate the confidence interval. Do this 10 times to simulate 10 SRSs with their 10 confidence intervals. How many of the 10 intervals captured the true mean μ? How many missed?



Of the 10 intervals that were calculated, 6 captured the true mean μ. 4 of them missed.

b) Reset the applet and click "Sample 25" to get the confidence interval from 25 SRSs. How many hit?



There were 21 out of 25 hits.

c) Click "Sample 25" repeatedly and write down the number of hits each time. Record the percent of hits for the following 100, 200, 300, 400, 500, 600, 700, 800, 900, 1000 SRSs. Comment on the percent of hits for the different SRSs.

Number of SRSs	**Percent of Hits**	**Number of SRSs**	**Percent of Hits**
100	$\frac{83}{100} = 83\%$	600	$\frac{493}{600} = 82\%$
200	$\frac{168}{200} = 84\%$	700	$\frac{570}{700} = 81\%$
300	$\frac{251}{300} = 84\%$	800	$\frac{656}{800} = 82\%$
400	$\frac{336}{400} = 84\%$	900	$\frac{730}{900} = 81\%$
500	$\frac{414}{500} = 83\%$	1000	$\frac{808}{1000} = 81\%$

The more SRS that were run, the close the data got to 80%. The sample started at around 83% hits and never was more than 84% or less than 80%. If I were to redo these runs, there may be times where the percent of hits is less than 80% (like in part a), but eventually, it will even itself out and converge towards 80%.

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2. An NHANES report gives data for 654 women aged 20 to 29 years. The mean BMI for these 654 women was $\bar{x}$ = 26.8. On the basis of this sample, we are going to estimate the mean BMI μ in the population of all 20.6 million women in this age group. We will assume that the NHANES sample is a SRS from a normal distribution with known standard deviation σ = 7.5

a. Construct three confidence intervals for the mean BMI μ in this population using 90%, 95%, and 99% confidence.

The formula for confidence interval is: $CI = \bar{x} \pm z_{\alpha/2}\frac{\sigma}{\sqrt{n}}$. We know that $\bar{x} = 26.8, n = 654, \sigma = 7.5$.

For 90% confidence, we know that $\alpha = 1 - .9 = .1$, so $\frac{\alpha}{2} = 0.05$. Using the critical values table, we know that $z_{0.05} = 1.645$. So, we can construct our confidence interval:

$$\bar{x} - z_{\alpha/2}\frac{\sigma}{\sqrt{n}} < \mu < \bar{x} + z_{\alpha/2}\frac{\sigma}{\sqrt{n}}$$

$$26.8 - 1.645\frac{7.5}{\sqrt{654}} < \mu < 26.8 + 1.645\frac{7.5}{\sqrt{654}}$$

$$26.318 < \mu < 27.282$$

For 95% confidence, $\alpha = 1 - .95 = 0.05$, so $\frac{\alpha}{2} = 0.025$. Using the critical values table, we know that $z_{0.025} = 1.960$. Thus, our confidence interval is:

$$26.8 - 1.96\frac{7.5}{\sqrt{654}} < \mu < 26.8 + 1.96\frac{7.5}{\sqrt{654}}$$

$$26.225 < \mu < 27.375$$

For 99% confidence, $\alpha = 1 - .99 = 0.01$, so $\frac{\alpha}{2} = 0.005$. Using the critical values table, we know that $z_{0.005} = 2.575$. Thus, our confidence interval is:

$$26.8 - 2.575\frac{7.5}{\sqrt{654}} < \mu < 26.8 + 2.575\frac{7.5}{\sqrt{654}}$$

$$26.045 < \mu < 27.555$$

b. What are the margin of errors for 90%, 95%, and 99% confidence?

The margin of error comes from the $z_{\alpha/2}\frac{\sigma}{\sqrt{n}}$ part of the confidence interval.

For 90% confidence, our Margin of Error is $E = 1.645\frac{7.5}{\sqrt{654}} \approx 0.482$.

For 95% confidence, our Margin of Error is $E = 1.96\frac{7.5}{\sqrt{654}} \approx 0.575$

For 99% confidence, our Margin of Error is $E = 2.575\frac{7.5}{\sqrt{654}} \approx 0.755$

c. How does increase in confidence level change the margin of error when the sample size and population standard deviation remain the same?

Increasing the confidence level, while keeping the sample size and population standard deviation the same increases the Margin of Error. This is because a higher confidence level means that we want to be even more sure that our population mean, μ, is inside that interval, so it requires us to widen the interval.

d. Suppose we have a SRS of just 100 young women. What would be the margin of error for 95% confidence?

For 95% confidence of a sample size of 100, our Margin of Error is $E = 1.96\frac{7.5}{\sqrt{100}} = 1.47$

e. Find the margins of error for 95% confidence based on SRSs of 400 young women.

For 95% confidence of a sample size of 400, our Margin of Error is $E = 1.96\frac{7.5}{\sqrt{400}} = 0.735$

f. Find the margins of error for 95% confidence based on SRSs of 1600 young women.

For 95% confidence of a sample size of 1600, our Margin of Error is $E = 1.96\frac{7.5}{\sqrt{1600}} = 0.3675$

g. Compare the margins of errors. How does increase in sample size change the margin of error when the confidence level and population standard deviation remain the same.

Increasing the sample size while keeping the confidence level and population standard deviation the same decreases the Margin of Error. This is because a larger sample size gives a more accurate estimate of the population mean, so in turn, we will need a smaller interval (smaller margin of error).

This part of the reflection presents data on employees' satisfaction of a company. The information came from a sample of 15,000 employees. The reflection is of two parts. The first part deals with hypothesis testing of mean from a single sample and the second part is on a single proportion. Conduct the required statistical analysis and answer the questions that follow.

Part 2

If the level of employee satisfaction drops below 0.60 overall, then there is a belief that there may be a serious problem with morale in that department. There have been rumors that the

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Human Resources department (hr) may be having just such issues. Using a statistical package, test to determine if the mean employee satisfaction level in the Human Resources department is less than 0.60. The data set for this analysis is saved under Week V in the content area of D2L. Click on this link to open the data set.

1. Is satisfaction level a qualitative or quantitative variable?

For this dataset, satisfaction is given as a percent, making it a quantitative variable, due to it being measured on a ratio scale.

2. Graph the employee satisfaction level for the **Human Resources department (hr)** with an appropriate graph and calculate statistics appropriate for this type of data.

Total Number of HR Employees (Sample Size)	$n = 739$
5 Number Summary	
Minimum	0.09
Q1	0.43
Median	0.61
Q3	0.81
Maximum	1
IQR	0.38
Other Statistics Needed	
Sample Mean	$\bar{x} = 0.5988$
Sample Standard Deviation	$s = 0.2479$

3. Conduct the appropriate hypothesis test using the following steps.
   a. Determine and state the null and alternative hypotheses.
      Null--H_0: $\mu \geq 0.6$, There are no morale issues in the HR department.
      Alternative--H_a: $\mu < 0.6$, There is a morale problem in the HR department.

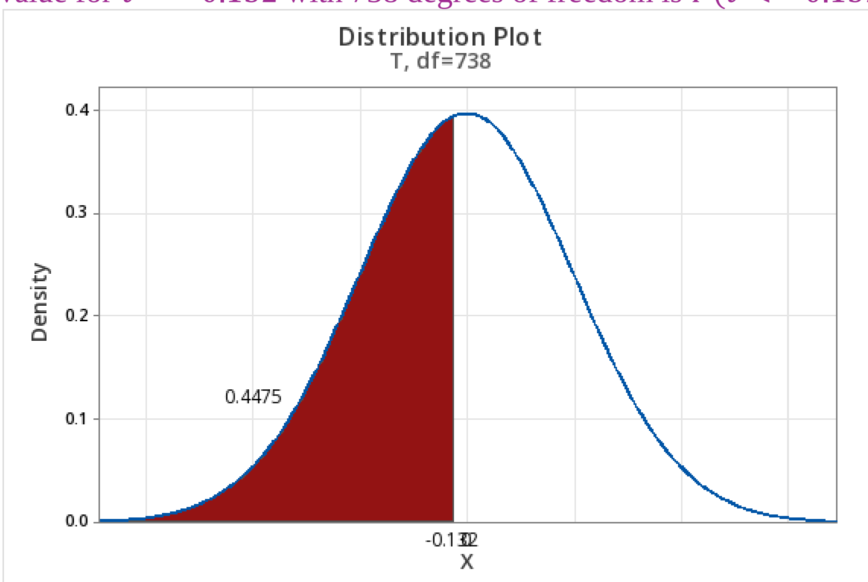
   b. Use a significance level of α = 0.05.
   c. Identify the appropriate test statistic, and compute its value. Why is this the appropriate test statistic?
   Because we do not know the standard deviation of the population, we must use the t-test to determine our test statistic. To calculate our test statistics, we will use the formula: $t_0 = \frac{\bar{x}-\mu}{s/\sqrt{n}}$

$$t_0 = \frac{0.5988 - .6}{0.2479/\sqrt{739}} = \frac{-0.0012}{0.009119} \approx -0.132$$

   d. Validate the assumptions of the hypothesis test.
   There are 4 assumptions that we made about this data:
   1. The data is quantitative.

   a. This is validated, because as in Question 1, we determined that satisfaction was a quantitative variable
2. The data is obtained via a random sample of size n.
   a. This is validated, because we used the random sample of 15,000 employees to obtain the data.
3. The population is normally distributed or the sample size n is large, $n > 30$.
   a. This is validated because we have a sample size of $n = 739$, clearly greater than 30.
4. The population standard deviation σ is unknown.
   a. This is validated, because in our population of employees at this company, we are not given the standard deviation.

e. Determine the P-value.
To determine the P-value, I used the tcdf function on my TI-84.
The P-value for $t = -0.132$ with 738 degrees of freedom is $P(t < -0.132) \approx 0.4475$.



f. Make a decision to reject or fail to reject the null hypothesis, H0.
We are going to **fail to reject the null hypothesis**, H_0.

Because H_a is looking for values less than, we know that we are dealing with a left-tailed curve. In order to reject the null hypothesis, $Pvalue < \alpha$. Because $0.4475 \geq 0.05$, we fail to reject the null hypothesis.

h. State the conclusion in terms of the original problem.

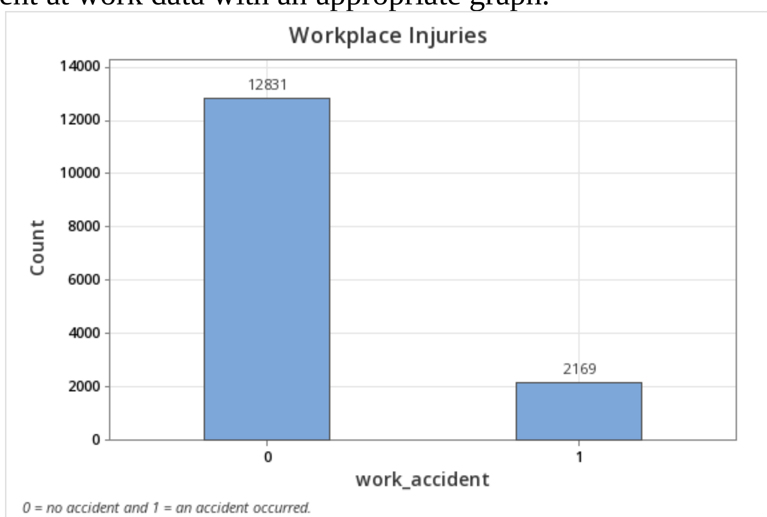
There is not sufficient evidence at the 0.05 level of significance to conclude that the HR department has a morale problem.

4. Based on our conclusion from the previous step, what type of error could we have just made (Type I or Type II)? State the practical implications of this error.

We could have made a Type II error, failing to reject a null hypothesis that is really false. The practical implication of committing this error would mean that there is in fact a morale problem in our HR department. This could mean that we lose a lot of HR employees, and could impact other areas of the company.

5. Would it be appropriate to compare this test to a confidence interval for the mean? Why or why not?
Yes, it would be appropriate to compare this test to a confidence interval. By constructing a confidence interval for the satisfaction level, we can assess whether the hypothesized value of 0.60 falls within the interval. If the interval includes the value of 0.60, it would support our conclusion that there is not enough evidence to suggest morale issues in the HR department. However, if the interval does not include 0.60 and is in fact lower, it would let us know that we may have made that Type II error and would let us know that we need to further investigate the data.

Part 3
According to the US Bureau of Labor Statistics, there was a 12.4% incidence of workplace injury in 2016 in the private sector. If the workplace accident rate is above 12.4%, the company will increase inspections and implement additional safety training. For this data, is there statistical support to increase inspections and implement additional safety training? Use the entire dataset and the work accident variable where 0 = no accident and 1 = an accident occurred.
1. Is work_accident a qualitative or quantitative variable?
Work_accident is a qualitative variable because we are representing a category (accident or no accident).
2. Graph the accident at work data with an appropriate graph.



3. Calculate statistics appropriate for this type of data.

Total Number of Employees (Sample Size)	$n = 15000$

Number of employees who have never had an accident	12831
Employees who have had an accident occur	$x = 2169$
Proportion in the sample who have had an accident occur	$\hat{p} = \frac{2169}{15000} = 0.1446 = 14.46\%$

4. Conduct the appropriate hypothesis test using the following steps.

a. Determine and state the null and alternative hypotheses.

Null--$H_0: p \leq .124$, The percentage of workplace accidents is at or below 12.4%.

Alternative--$H_a: p > .124$, The percentage of workplace accidents is above 12.4%.

b. Use a significance level of α = 0.05.

c. Validate the assumptions of the hypothesis test.

There are 3 assumptions that we made about this data:

1. The data is categorical.
    a. This is validated, because as in Question 1, we determined that satisfaction was a qualitative variable
2. The data is obtained via a random sample of size n.
    a. This is validated, because we used the random sample of 15,000 employees to obtain the data.
3. The sample size is sufficiently large that the sampling distribution of sample proportion is approximately normal such that $np > 5$ and $n(1 - p) > 5$
    a. This is validated because with $n = 15000$ and $p = .124$, $np = 15000(.124) = 1860 > 5$, and $n(1-p) = 15000(.876) = 13140 > 5$.

d. Identify the appropriate test statistic, and compute its value.

Because we are dealing with population proportions, we are finding the critical z-value. Because we do not know the standard deviation, we must find σ_p̂ using the formula $\sigma_{\hat{p}} = \sqrt{\frac{p(1-p)}{n}}$. To calculate our test statistics, we will use the formula $z = \frac{\hat{p}-p}{\sigma_{\hat{p}}}$.

$$\sigma_{\hat{p}} = \sqrt{\frac{p(1-p)}{n}} = \sqrt{\frac{.124(1-.124)}{15000}} \approx 0.00269$$

$$z = \frac{\hat{p} - p}{\sigma_{\hat{p}}} = \frac{.1446 - .124}{0.00269} \approx 7.66$$

e. Determine the P-value.

To determine the P-value, I used the normalcdf function on my TI-84.

The P-value for $z = 7.66$ is $P(z \geq 7.66) = P(z \leq -7.66) \approx 0$.

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f. Make a decision to reject or fail to reject H0.
We are going to **reject the null hypothesis**, H_0.

Because H_a is looking for values greater than, we know that we are dealing with a right-tailed curve. In order to reject the null hypothesis, $Pvalue < \alpha$. Because $0 < 0.05$, we reject the null hypothesis

g. State the conclusion in terms of the original problem.
There is sufficient evidence at the 0.05 level of significance to conclude that the percentage of workplace accidents is above 12.4%. Thus the company should implement increased inspections and additional safety training.

4. Based on our conclusion from the previous step, what type of error could we have just made (Type I or Type II)? State the practical implications of this error
We could have made a Type I error, rejecting the null hypothesis when the null hypothesis is true. The practical implication of committing this error would mean that the percentage of workplace accidents is at or below 12.4%. This would mean that the company spent more money and time dealing with increased inspections and implementing additional safety training when it was not needed.