

Comprehensive Question #1 Who Decides About What is Taught and How It is Taught?

A teacher named Bill Taylor previously worked in a very conservative district. His new district was more to his liking because he felt free to address societal issues and problems, even controversial ones, in an honest way. He felt it important for his students to understand the many “sides” of an issue or problem. To ensure that all the sides of an issue or problem emerged and were considered he sometimes felt it was necessary to play devil’s advocate.

The principal, Pat Summers, recently came to Bill’s room for an observation of a social studies lesson. The subject of the lesson was the four freedoms described by President Franklin Roosevelt in his State of the Union speech on January 6, 1941. The freedoms he enumerated were the freedom of speech and expression, the freedom of every person to worship God in his own way, the freedom from want, and the freedom from fear. As the lesson unfolded, many controversial topics were discussed, often heatedly. Some of the topics included whether it was appropriate to say a prayer in school, whether it is immoral and illegal to burn the US flag, whether citizens had the obligation to care for others when they seemed unwilling to work and to support themselves, and whether it was acceptable to have a member of a “radical” group like the Ku Klux Klan or the Aryan Brotherhood or the Black Panthers speak out against other races or groups. Bill saw to it that all sides of these issues were considered.

During the pre-conference, Bill and Pat had agreed that Pat would focus on lesson design, student motivation, and content to stimulate student thinking. During the lesson, Pat became uncomfortable with some of the content. He noted that Bill was working hard at getting his students to think and to examine their values. The discussion in the post conference focused on the students' ability to think and examine their values. The substance of the lesson was not discussed.

The next week Pat Summers received an angry complaint from Mr. Riley, the commander of a local veteran’s organization, who demanded an investigation into the subject matter Bill Taylor was teaching his students. Mr. Riley’s daughter had reported that the teacher was very critical of the government and had cast scorn on the expression of patriotism.

Pat Summers told Mr. Riley that the matter would be investigated, but the parent did not appear to be satisfied with that promise. “I’ll call you again on Friday to see what steps you’ve taken to correct this deplorable situation,” he said. Then he added, “You should be aware that there are others in the community who are also concerned about this situation and who will not sit back and allow our children’s attitude toward our country to be poisoned by malcontents teaching in our schools!”

Your task for the comprehensive question: You are to assume the role of the teacher, Bill Taylor, in this situation. You are to identify the issues, problems, and opportunities associated with the scenario and describe how you will handle them when confronted by the principal and/or community members.

Response:

If I were Bill Taylor in this situation, I would recognize that there are some serious concerns here. I believe that it is important for teachers to help students become thoughtful, engaged participants in our society. This means giving them opportunities to explore and engage in complex issues, consider multiple perspectives of an issues, and develop skills to discuss controversial topics respectfully and thoughtfully.

To help prevent misunderstandings or backlash before they arise, I would take a few proactive steps. First, I would clearly communicate my goals and instructional approach to families early in the year and continue to communicate with them often. I would explain that my classroom will be a space where students will be pushed outside of their comfort zone. We may work through topics that can be controversial, but we will explore all sides of each issue, looking at historical context and various viewpoints.

Second, I would notify families in advance when a lesson might involve particularly sensitive content. I think the most important part of this communication will be explaining the standards we are covering, as well as the learning objectives. This might help build trust with families that might feel that teachers are trying to “indoctrinate” students to specific viewpoints. By describing exactly why we are covering specific topics, it may help families to feel as though we are more in partnership in their student’s education.

In my classroom itself, this should not be the first time students are hearing about and discussing the information. I would ensure that students have the historical background, vocabulary and context that they would need before we engage in the discussion. For example, before the discussion on the four freedoms, we would examine

the historical context behind the speech, including World War II, what society was like at the time, and if there were specific threats to these freedoms. I would have students explore primary sources, including other people's perspectives from all sides of the issue. I would ask students to ground their arguments with the sources we are studying, rather than focusing on their own feelings.

This would also not be the first time that we would have held discussions in class. We would have practiced with lower stakes topics and focused on building skills and confidence. We would have, as a class, established group norms for discussions, including how to actively listen, ask clarifying questions, and how to support your views with evidence. Before each discussion, students and I would revisit our norms and remind ourselves how to disagree with one another respectfully. Students would have worked with topics and know that I as a teacher will often play that "devil's advocate" role to help push their thinking. Over time, they will become more comfortable with helping them to see all sides of an issue.

Now, assuming all these things were put in place, and Mr. Riley had still complained, there are few things that I would do. First, I would make sure to request a meeting with my principal to address the situation directly. In that conversation, I would explain the rationale for the lesson, including the standards we are covering, and how it aligns with learning objectives. I would also give an outline of the work we had done to prepare.

I would also ask my principal for honest feedback, whether it be good or bad, on the content of the lesson. I would ask for suggestions on how to possibly frame discussions of this type in the future. I would want to make sure to reinforce that my goal

for this lesson was not to be critical of the government, but to help students examine and explore all sides of the issue. Additionally, I would express my hope that the administration will support me in creating space for this type of dialogue with students and support student inquiry and not rush to appease parents. It is important for me to be open to feedback, while staying firm in my belief that having students explore all sides of these topics is important to help them grow.

If I were asked to meet directly with Mr. Riley, or, if he reached out to me, I would make sure to have the principal or another administrator present to help mediate the conversation. I understand that this is a topic that many people can become extremely passionate about, so having a third party there would help to ensure that the conversation remains productive and respectful. Throughout this discussion, I would listen carefully to Mr. Riley's concerns, clarify any questions or misconceptions he might have, and explain the overall purpose and structure of the lesson. Again, I would emphasize that my goal is to support students in becoming thoughtful, informed citizens—not to influence their personal beliefs.

It may be that Mr. Riley still walks away from this conversation unsatisfied with the outcome. Even so, I would be confident that I had approached this situation with professionalism, integrity and transparency. My work before this lesson, through communication with families, careful planning and preparation with students, will hopefully help to mitigate these types of issues. Still, I recognize that there will always be people who do not approve. This may even be an issue to cover with students in the future: what are schools and teachers allowed or not allowed to teach? Having these

conversations openly with students can help them to understand why education is important in our society.

Overall, I still believe that it is the responsibility of teachers to help students become thoughtful, engaged members of society. While we should approach controversial topics with care and preparedness, we should not avoid them. These will be things that students encounter outside of school. If the goal of education is to prepare students for society, we must allow for them to explore these issues with guidance and purpose. Careful planning can reduce conflict, but it is the courage to engage in these conversations that helps our students grow. That is the kind of teaching that I hope to inspire in my students.

Comprehensive Question #2

Provide an overview of your eFolio, sharing a few of your proudest learning moments, in an abbreviated form. Why did you choose these items? How do you see them helping to increase student achievement? Do you see components /artifacts in your eFolio as beneficial to teachers and students? Provide rationale to support your response.

Response:

While reviewing and working through completion of my eFolio, it is clear to me how much my skills as both a mathematician and an educator have grown throughout my time in this program. A common theme I saw throughout my eFolio was my commitment to improving mathematical thinking and problem-solving in my classroom. Whether through improving my own proof-writing, integrating purposeful technology, or designing differentiated learning paths, I have grown in my ability to support all students in becoming confident, curious thinkers.

One of my proudest areas of growth was in my own knowledge of mathematical reasoning and proof writing, especially throughout my math classes. In my undergraduate work, proofs were really something that I struggled with, even leading me to wonder if I should continue pursuing mathematics as a career path. However, throughout my courses, especially Abstract Algebra, Graph Theory, Topology, and Combinatorics, I know that I have become a much better writer and reasoner. As I continue to write proofs, I have learned how to best break down a problem and process the information given. I have become better about using precise mathematical language, being careful about assumptions that I was making, and even identifying the best way to prove a specific problem. I also have learned how to revise my work when something is incorrect.

Whether it be thinking through all possible cases to find the missing case, finding the gap in logic, or making sure that all conditions were satisfied before using a specific theorem,

proof-writing has helped me to slow down and clearly communicate my reasoning. This is something that I have carried over into my teaching. I have spent more time working with students on explaining why something works in math and showing the importance of justifying their steps. I am better at modeling how to break down a problem and to explain why a specific thing must be true. Through my use of examples and counterexamples, students are better able to articulate why specific theorems and ideas work in specific situations.

Throughout Math 515, one of the most valuable takeaways was that just adding in a piece of technology does not mean that students will be more engaged or better learn the material. It must be selected purposefully, giving attention to mathematical fidelity, how well it represents the mathematics and cognitive fidelity, and how well students actually think and learn. This class, being math specific, really gave me the opportunity to dive down deep with my peers and explore all of the cool technology tools that we could bring into the classroom. Every time I created a new project in the course, I could immediately envision how I would use it in my own classroom.

Two of the artifacts that I chose for this course really show the integration of technology and problem solving. First was my Desmos Activity Builder assignment on simplifying square roots. This lesson includes interactive, self-checking features, group discussion prompts, and even has students working through error analysis and justifying their steps. By giving students instant feedback, they are able to move at their own pace, while still being able to verify that they understand the material. I have used this artifact in my classroom, and students really enjoyed it. My second artifact was my own project on recreating a picture in the Desmos graphing calculator. This was a project that was

really fun for me and helped remind me of the constraints of different types of functions. I feel that using this project with students will push them to apply what they know about functions in a creative, open-ended way, promoting both algebraic fluency and visual reasoning. These projects demonstrate how technology can help students construct their own meanings in math.

A third class and moment that I was proud of comes from my work in ED 632: Curriculum Instruction and Learning Theory. This course really impacted my approach to differentiation and instructional design. I was really proud of my research that I did with the research informing teaching assignment. I wanted to become better at questioning with my students beyond just asking, “How did you do that?”, and instead push my students to reflect and justify their thinking at a deeper level. Strategies such as teacher press and follow-up questioning are things that I have implemented in my classroom. I now think more critically about how to frame my questions to promote reasoning and help students to formulate new ideas. Students are doing better about listening to other students and linking their thoughts to their classmates. That shift in classroom dialogue is something I am proud of, and it stemmed directly from the work I did in this course. Another thing from this course that I have since implemented into my class is the ideas on pathways plans. After a lesson, students need to take a short 2 question quiz to show that they have mastered the material. If they do not pass, it lets both them and me know that they need to go back to review. This system has helped support students self-monitoring their own knowledge, while giving me frequent check-ins with them.

I know that all of the work that I have done throughout this program will continue to influence my teaching for years to come. I have grown more confident in my ability to

engage students in meaningful mathematical thinking, to ask better questions and differentiate instruction, all while becoming a better mathematician myself. My eFolio demonstrates my growth, and celebrates my achievements, and it is something that I am truly proud of.

Comprehensive Question #9 Mathematics Education

Students are to select one of the following questions and respond completely:

- I) You are asked to develop a district plan to integrate problem solving into all secondary mathematics courses. In your plan you are to include the following:
 - a. Explain how heuristics, control, and metacognition each contribute to successful mathematical problem solving.
 - b. Discuss the similarities and differences between teaching mathematical problem solving and teaching mathematics through problem solving.
 - c. Provide a specific example of a lesson that gives students the opportunity to engage in problem posing in a meaningful way.
 - d. Expound how technology can be used as a tool to facilitate student problem solving.
- II) You are assigned to teach a dual credit math course next year. Utilize research on how students learn mathematics as it relates to neuroscience and mathematical mindsets to delineate how this shapes your teaching of mathematics for college credit. Outline how self-regulated learning can be included in your dual credit math course. Explain how you will approach the teaching of procedural and conceptual understanding in your college in the classroom course.
- III) As part of your research/action research for your thesis, you will be developing a precise problem statement, the purpose of your research and some research questions. Based on the purpose of your study:
 - a. State the research questions that will guide your research.
 - b. Formulate statistical hypotheses that you will test to answer your research questions.
 - c. Discuss the research design that will guide your data collection and justify the appropriateness of the chosen design.
 - d. What kind(s) of data will you need to collect to test the hypotheses you formulated?
 - e. What will be the independent and dependent variable(s) of a statistical model you will develop?
 - f. Discuss the statistical test(s) you will perform to test your hypothesis, including making sure that the assumptions are met?
 - g. Discuss how the results of the statistical analysis will be interpreted.

Response (Prompt #1):

I chose prompt 1, on developing a district plan to integrate problem solving into secondary math courses. Problem solving is a huge aspect in mathematics, but as educators, we often teach students formulas, without focusing on how to help them develop problem solving strategies. It is a way that teachers can help students make sense of unfamiliar situations, apply knowledge, and think critically about patterns and structures. By intentionally working towards incorporating problem solving into all courses, educators can push students to become confident mathematicians.

An effective problem-solving curriculum should include three specific components. These components: heuristics, control, and metacognition, will help students to become better problem solvers. The first component is heuristics. Heuristics are strategies and methods that students may use to help them solve problems. They are not explicit algorithms, but rather general ideas that can help. Examples include methods such as drawing a diagram, making a table, working backwards and looking for patterns. Often times, these strategies can help students to find a foothold in a tricky problem. While students might not be able to immediately solve a problem using only using heuristics, these tools can help students to make sense of an idea and find a path forward.

The second component is control. Control is the decision-making process that helps students to make strategic decisions during problem-solving. It involves being able to select an appropriate strategy, monitoring its effectiveness, and knowing when to change to a new strategy. Students with strong control are able to pause and reflect on their progress and manage their time and effort effectively. Teaching control means

helping students to become more strategic, working with students to think not just about what they are doing, but why they are doing it.

These ideas lead directly into the third component, metacognition. Metacognition is the ability to think about one's own thinking. It plays a vital role in helping students become independent problem solvers. When students engage in metacognitive practices, they are more likely to ask themselves meaningful questions and self-reflect on their habits. This helps them to manage their thought processes and, like control, know when to move on to a new strategy. To help students with metacognition, teachers can model reflective thinking aloud, encourage students to journal their thoughts throughout problem solving, and provide time and prompts for students to reflect on their process after completing a task. This helps students think about their thinking, which will help to strengthen their problem-solving abilities.

Along with these three components, it is important to consider how problem solving is taught in the classroom. In particular, there is a distinction between teaching mathematical problem solving and teaching mathematics through problem solving. While both of these approaches help students to learn mathematics and become better thinkers, there is an important distinction. Understanding the difference allows for educators to incorporate both methods into their instruction.

Teaching mathematical problem solving involves explicitly instructing students in the process of solving problems. When teaching mathematical problem solving, we tend to focus on the three components mentioned above, and work with students on building strategies to approach problems. For example, a lesson might focus on how to break a

word problem into smaller steps, or work with students on understanding how to use various heuristics. The problems are not the end goal—the primary focus is to build problem-solving skills and habits.

On the other hand, teaching mathematics through problem solving involves using mathematical problems as the starting point for instruction. Instead of beginning with formulas or procedures, students use a task to help them develop the underlying concept. Through exploration, students are exposed to new mathematical ideas. This helps them to develop a deeper understanding and create connections on how mathematics shows up in the real world. For example, rather than directly being told how to solve a system of equations and then being given a word problem to test their knowledge, the lesson might start with having students determine how many of each type of coins are in a bag, but they must add up to a specific amount and there are a specific number. Through this process, students naturally develop the need for and understand various methods of substitution or elimination.

While the two approaches are different, both are important for student learning. Teaching mathematical problem solving helps to strengthen strategic thinking and build skills, while teaching mathematics through problem solving helps make learning more meaningful for students. Both should be used in tandem with each other to help students grow in their mathematical abilities.

An effective way to help students become more independent and creative mathematical thinkers is to give them opportunities to not only solve problems, but to pose their own questions.

One example lesson that incorporates problem posing could be in an Algebra class, in which we are studying linear equations with one unknown. The teacher would start by presenting the following situation to students:

Student A can mow the lawn in 1.5 hours. Student B can mow the same lawn in 45 minutes.

Instead of providing a specific question to solve, students are asked the following prompt: “What kinds of mathematical questions could we ask based on the situation? Provide at least 3 different questions that we could ask. If you want to add additional constraints to the problem, you are able to.” Then students will work in small groups to brainstorm possible questions that they could investigate. Some questions that students might ask include:

- How long would it take if they worked together?
- Student A starts and mows for 15 minutes by themselves, and then is joined by Student B. How long does it take them to finish the job together?
- Student B mows for 20 minutes and then quits. The job is then finished by Student A. How long did it take to finish mowing the lawn?
- The students started mowing together. After 30 minutes, student B has to leave. How much of the lawn is left to mow?
- The students work together to mow the lawn. They are paid 50 dollars. How much should each student get paid?

After students have brainstormed a variety of questions, students will work together to create a solution for their problems. They will work to model and solve it using rational equations, diagrams, and tables.

This type of lesson allows for students to take ownership of the problem-solving processes. When students are constructing their own problem, they are thinking deeper, automatically differentiating to their own knowledge, and forcing themselves to think critically. It also naturally reinforces the algebraic concepts we are studying, as they will have to use those concepts to help them solve their chosen problem. When students pose their own questions, they tend to feel a greater connection to the mathematics.

Technology is another great way for introducing and enhancing problem solving in mathematics courses. It can help to make mathematics more interactive and visual, while providing exploration that can help deepen and construct understanding. Tools like Desmos and GeoGebra allow students to manipulate variables and see immediate changes in graphs or geometric figures, which can help in exploration of ideas. For example, by using a slider for slope in the Desmos graphing calculator, students can immediately see the differences between linear equations with positive and negative slopes.

Technology can also help for student collaboration. Platforms like Desmos Classroom, Padlet, or even Google Docs, will allow for students to work together, share strategies, and reflect on their thinking. It is easy to see how students are approaching problems and provide them immediate feedback. Desmos Classroom is particularly effective for this type of work. It allows for teachers to see each student's screen, take

screenshots of student work to share, and can give interactive feedback to students to help them push their thinking further. When used with intention, technology is a great way to enhance problem solving.

Incorporating problem solving across all secondary mathematics courses is essential for preparing students to think critically and apply mathematics beyond the classroom. To make this a consistent part of instruction, teachers should aim to incorporate at least two problem-solving tasks a month. Additionally, teachers should regularly embed opportunities for students to pose problems, reflect on their learning, and practice a variety of problem-solving strategies. A district wide commitment to this kind of instruction should be supported through professional development, shared curricular resources, and structured collaborative time across the department. With these supports in place, we can push students to help make sense of the world, think critically, and grow into confident mathematicians.